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Survey of Time-Optimal Attitude Maneuvers

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Introduction

PRECISION pointing control and rapid maneuvering capabilities have long been part of many space missions; examples include military observation platforms, communications satellites, and exploratory space missions. Rapid retargeting may be an intrinsic part of the mission profile, as in military and exploratory applications, or it may be required to correct, periodically, the guidance and navigation sensors of a spacecraft. Consequently, research in attitude maneuvers and time-optimal controls has been a consistently strong field of study. Of particular interest is the time-optimal attitude maneuver; this problem combines aspects of many different fields, such as mathematics, optimal control theory, spacecraft dynamics, elasticity, and structural mechanics. In the study of attitude maneuvers and control, researchers have investigated the response of satellite control systems to disturbances caused by the influence of solar pressure, gravity gradients, the effectiveness of active control devices, and the best placement of actuators and sensors on a given structure. Numerous studies have been conducted for specific satellite configurations to determine a time-optimal attitude control for a predetermined maneuver. Further, the mathematical procedures required to investigate this problem have inspired a number of papers on techniques to address the difficulties which arise from time-optimal control. This survey reviews the literature of the last thirty years, including many of the more recent advances in the field, covers the aforementioned topics as well as specific attitude maneuvers, and traces the development of time-optimal and near time-optimal control algorithms.

We begin by considering the formulation of the time-optimal control problem and the difficulties faced in obtaining the solution. The equations for the rotational motion of any rigid body (Euler's equations) are quite elegant and compact; they are also coupled and highly nonlinear. In addition, the high angular velocities of time-optimal maneuvers will cause gyroscopic stiffness through the nonlinear terms. Euler's equations are expressed as

$$\begin{aligned} I_1\dot{\omega}_1 + (I_3 - I_2)\,\omega_3\omega_2 &= u_1 \\ I_2\dot{\omega}_2 + (I_1 - I_3)\,\omega_1\omega_3 &= u_1 \\ I_3\dot{\omega}_3 + (I_2 - I_1)\,\omega_2\omega_1 &= u_1 \end{aligned} \tag{1}$$

where u_i is the control torque, ω_i is the angular velocity, and I_i is the moment of inertia about the ith principal axis.

These equations completely describe the rotational motion of a body with respect to an inertial frame, but they are not sufficient to determine the attitude of the spacecraft. Because the problem under investigation involves attitude maneuvers of a satellite, additional equations that yield the instantaneous attitude of the vehicle must be used. The two most common representations of the orientation of a body are Euler angles and Euler parameters (quaternions). Euler angles have numerical singularities for certain attitudes whereas Euler parameters are mathematically well-behaved; the latter are defined in terms of Euler's Principal Rotation Theorem

$$\beta_0 = \cos\phi/2 \qquad \beta_i = l_i \sin\phi/2 \qquad i = 1, 2, 3 \tag{2}$$

where l_i are the direction cosines for the Euler axis, and ϕ is the angle of rotation about that axis. The Euler parameters are time-dependent due to the rotation of the body, and it can be shown that

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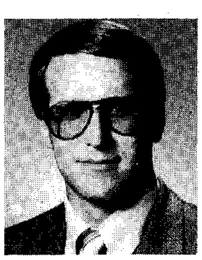

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this variation with time is governed by the nonlinear ordinary differential equations

$$\begin{pmatrix} \dot{\beta}_0 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{pmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_1 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \tag{3}$$

Closed-form solutions of the complete set of equations exist for only a handful of cases involving simple rotations or simple geometry of the body. For example, the solutions for rotation about a single axis ($\omega_1 = \omega_2 = 0$), or torque-free rotation of an axisymmetric body ($I_1 = I_2$) are well-known.

The time-optimal, multi-axis attitude maneuver adds additional difficulties to the problem because of the time-optimal nature of the maneuver. Simply stated, we seek to minimize the time required to move the satellite from a specified initial attitude to a specified final orientation. The problem is very simple to formulate using methods developed in optimal control theory; the performance measure is the total maneuver time, t_f and the dynamic equations are constraints that the solution must satisfy. Mathematically, the problem is to minimize J where

$$J = \int_{t_0}^{t_f} \mathrm{d}t \tag{4}$$

subject to the constraints given by Eqs. (1) and (3). The Hamiltonian, composed of the integrand of the performance measure and the dynamic equations in state vector form, is defined to be

$$H = 1 + \underset{\sim}{\lambda}^T f(x, u) \tag{5}$$

where $\underset{\sim}{\lambda}$ is the costate vector (Lagrange multipliers), x is the state vector, and f represents the governing equations of motion given in Eqs. (1) and (3). Applying Pontryagin's necessary conditions, the equations that must be satisfied to obtain the time-optimal control problem are

$$\dot{x} = \frac{\partial H}{\partial \underset{\sim}{\lambda}} = f(x, u)$$

$$\dot{\underset{\sim}{\lambda}} = -\frac{\partial H}{\partial x} = \left[\frac{\partial f}{\partial x}\right]^T \underset{\sim}{\lambda} \tag{6}$$

$$H(x^*, u^*, \underset{\sim}{\lambda}^*) \leq H(x^*, u, \underset{\sim}{\lambda}^*)$$

where the ( )* quantities are the optimal trajectories.

The system given by Eq. (6) forms a two-point boundary value problem, with boundary conditions $x(t_0) = x_0$, $x(t_f) = x_f$, and a free final time; Eqs. (5) and (6) provide the necessary conditions for the time-optimal problem. These expressions are simple in appearance, but they have no known solution, numerical or analytical, except when certain restrictions are applied to the problem; the assumptions may be in the form of specific configurations, such as an inertially symmetric body, or restricted motions such as single-axis rotations about a principal axis. For example, when the problem is defined as a single-degree-of-freedom system with a single controller, the solution is the well-known bang-bang maneuver. The control profile is characterized by saturated controls (i.e., $|u| = 1$) for the entire maneuver, with at most, one switch.

As complicated as the system given by Eqs. (1) and (3–6) may be, it models only the rigid-body motion. Modern spacecraft make use of lightweight materials which are quite strong, but which are also quite flexible. Therefore, a number of researchers have incorporated flexibility effects in their state models. In addition to the rigid-body equations introduced earlier, the flexibility effects add theoretically infinite degrees of freedom to the problem. The motion of flexible appendages is an additional factor that must be addressed in current research, and the increased complexity in the optimal control formulation is difficult to overcome.

Because there are so many different topics addressed in time-optimal control problems, the remainder of this paper is divided into sections which progress from the publications that were instrumental in early formulations of the problem through more advanced solutions of time-optimal problems. The survey begins with papers which do not specifically address optimal controls, but which make a contribution to the field, particularly in the area of new and better models of spacecraft and their behavior. Included with the developmental papers are those which present mathematical or computational advances in solutions to the time-optimal problem. From there the survey moves on to time-optimal problems, beginning with the early attempts to address some very basic concepts. Rigid-body solutions to single-axis maneuvers are then discussed; these solutions form the basis for the more advanced topics such as flexible spacecraft or specific satellite configurations.

The final set of papers are those which have expanded the envelope of solvable systems in time-optimal controls. They are the most recent attempts to solve the full three-dimensional problem. No one has developed a rigorous solution for the general three-dimensional, time-optimal attitude maneuver, either for a rigid or a flexible system. The papers in this final section indicate that a numerical solution method probably does exist and is likely to be determined in the next few years.

Developmental Papers

Background Papers

The papers containing background information are extremely diverse. Therefore, they have been grouped rather loosely into three subcategories: development of flexibility effects, modification and development of equations of motion, and advances in control algorithms or solution techniques.

Flexibility Effects

The papers dealing with flexibility effects do not always include time-optimal controls, but they are included because they incorporate flexibility effects into different types of control problems. In a paper which has been frequently cited as a foundation for control of flexible structures, Breakwell[1] develops a closed-loop minimum control effort solution for a flexible body including the first n flexible modes in the system model. In a related note, Junkins[2] observed that control spillover could be reduced by penalizing the second time derivative of the control variables in the cost functional; smooth transitions in the control profile at the beginning and end of the maneuver were assured. Krabs[3] discusses the distributed control of a flexible system, approaching the problem from a rigorous, mathematical point of view. Gibson[4] explores the ramifications of truncating an infinite modal series to a finite number of modes and the conditions necessary to make sure that the control profile for the finite series representation does converge to the control of the true, infinite mode system. Many other papers deal with simple, single-axis maneuvers of flexible spacecraft. Barbieri et al.[5] develop the equations of motion and closed-loop control algorithms for a satellite which must target the optical rays off mirrors which are at the end of flexible manipulators. The control algorithm is not time-optimal, but the model and application to satellites is relevant to current systems. Oz and Meirovitch[6] develop a linear quadratic regulator (LQR) optimal closed-loop control law for a single-axis maneuver of a flexible system. The general form of an LQR cost functional has quadratic terms in the states and control variables as shown

$$J = \frac{1}{2} x^T(t_f) H x(t_f) + \frac{1}{2} \int_0^{t_f} (x^T Q x + u^T R u)\, \mathrm{d}t \tag{7}$$

where the weight matrices, Q and R, are chosen to achieve a certain convergence time and control magnitudes. The authors, utilizing the fact that the system is gyroscopic and symmetric in the equations of motion, uncouple the governing equations. The control loops employ decoupled observers, and the resulting system of equations consists of $n/2$ 2×2 matrix Riccati equations, rather than a single large matrix. This approach reduces the computational difficulties in working with a large system as well as the time to compute an acceptable solution. Turner and Chun[7] dealt with flexible systems using distributed controls and an LQR, rather than a time-optimal, algorithm. The computational approach is a single-stage continuation method on the two-point boundary-value problem (TPBVP). The initial costates are found by analytically solving a zero-order (linear) system. The system is then perturbed slightly, adding in small nonlinearities, and a new solution is determined using the zero-order costates as an initial guess. The magnitude of the nonlinear terms are increased again and a new solution is found, this time using the first-order solution as the starting point. This approach is also used by Turner and Junkins.[8] Bainum and Li[9] use an LQR for flexible systems which is not limited to a single-axis maneuver; by using a quasilinearization technique in conjunction with the method of particular solutions, they develop a control algorithm for a general three-dimensional maneuver.

Meirovitch,[10] Vadali,[11] and Junkins et al.[12] address the issues of stability of flexible systems using a Lyapunov stability criterion; Hablani and Jansz[13] and Skaar et al.[14] address the control of flexible members using actuators commonly available for spacecraft applications. In a paper which deals with actuator dynamics, Benninghof and Boucher[15] find that there is a tradeoff between maneuver time and complexity of the model. They show that, below a certain maneuver time, including more modes in the model results in a higher spillover energy, or residual vibration, at the end of the maneuver. This limiting time is shown to be related to the time it takes for information to travel between the two most widely separated actuators. Since the maneuver time for a time-optimal maneuver is, by definition, relatively short, actuator placement becomes a more important consideration in control implementation and in modeling the system response.

Equations of Motion

Several papers address the development of the equations of motion for a given satellite system. Hooker[16] chose to eliminate the torque constraints which produces a system of equations that are similar to those developed by the Lagrangian approach, but are easier to modify. Longuski[17] takes an interesting approach to the problem by approximating the solution to Euler's equations and the Eulerian angles for a near-symmetric spacecraft. Euler's equations are quasilinearized from a symmetry standpoint and then put through a change of variables such that the resulting set of equations are ordinary differential equations with constant coefficients and time-varying forcing functions. These equations are then solved in terms of Fresnel integrals. The Euler angle equations (a 3-1-2 rotation) are put through a similar change of variables including some small-angle assumptions, and are then approximated by an asymptotic series expansion to yield an approximate system response to a control input.

The use of Euler parameters has become quite common for the reasons explained earlier; however, the redundancy in the set of parameters produces an overdetermined system, but the constraint equation, which is nonlinear, cannot be used to eliminate the additional coordinate. Fortunately, Vadali et al.[18] have shown that the Euler parameter constraint leads to a transversality condition requiring the costates to be orthogonal to the Euler parameter vector. Vadali later[19] rigorously shows that in the Lagrangian development of the equations of motion, this constraint multiplier is zero; in time-optimal controls, the control profile is independent of the constraint multiplier. Therefore, the constraint does not need to be included explicitly in the formulation of the system.

Other studies have addressed some imposing problems in attitude dynamics and controls. Ho and Herber[20] examined the development of a set of equations for the simulation of flexible systems; Ho[21] extended this study to include flexible, multibody systems. Dwyer[22] modeled a rigid, asymmetric spacecraft and studied closed-loop control under minimum energy constraints; however, his equations of motion cannot be linearized through symmetry arguments. Quinn and Meirovitch[23] deal directly with the equations of motion for a flexible spacecraft where the actuator dynamics are included. The open-loop control profile for a rigid-body maneuver is developed and then it is implemented by distributing the control force such that the least vibrational excitation is induced.

Control and Model Development

In an early paper, Meyer[24] uses the Euler Principal Rotation as a measurement of system error to give a single feedback variable during a multi-axis maneuver. Wie et al.[25] use quaternion feedback gain matrices to perform the Euler axis manuever. They also explore the stability of the approach and the robustness with respect to uncertainty in the initial body rate and inertia. Long[26] uses the Principal Rotation Theorem for the maneuver, but uses the Euler axis as one of the axes of the reference frame. The maneuver is rest-to-rest, and the open-loop control forces the spacecraft to perform the Euler Rotation. The computational savings with this method are quite significant; the motion of the spacecraft is planar and the equations of motion are considerably simplified, although the reference frame is a nonprincipal frame. Gauvrit et al.[27] explore the open-loop controllability of time-optimal satellite maneuvers by transforming to the frequency domain, finding that the controllability criteria are easier to define in this domain. White et al.[28] discuss controller algorithms for general applications by addressing the analogy between rate-ledge relay and time-fuel optimal controllers. The rate-ledge relay is an empirically determined reaction control jet relay profile, with a sloping deadband. They show the parallel between the two types of control laws and the transformations between them.

Several authors address the issue of nonlinear control problems. Carrington and Junkins[29] chose to expand the closed-loop control variable in a polynomial and determine the coefficients by iteratively solving successive increasingly nonlinear systems. Another approach, taken by Dwyer,[30] employs a nonlinear coordinate transformation to obtain a linear system in the transformed space. Linear closed-loop control algorithms are used to solve the transformed system, and the resulting solution is put through the inverse transformation to arrive at a solution to the nonlinear system. Chowdhry et al.[31] explored the open-loop control of angular velocity (but did not include attitude control) in the absence of direct control over one of the angular velocity components. Skaar and Kraige[32] used an open-loop optimal power criteria to develop a control algorithm for a general three-dimensional maneuver.

The remaining papers in this section minimize control effort; that is, the cost functional is of the form

$$J = \frac{1}{2}\int_0^{t_f} \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u}\, \mathrm{d}t \tag{8}$$

and although the focus of each paper is the same, the problems they investigate are very different. Ioslovich[33] addresses the configuration of two open-loop controllers with unequal moment arms. Junkins and Turner[34] investigate multi-axis maneuvers by using the closed-form solution of the single-axis maneuver as a starting point and obtaining the open-loop solution by successive relaxation of the final boundary conditions. Vadali and Junkins[35] used yet another technique, suggested for use in flexible systems, of including the second time derivative of the controls in the cost functional. However, in this case, the purpose was to achieve optimal momentum transfer, rather than control of vibrations. The last paper on optimal control effort to be included with this group is also by Vadali and Junkins,[36] in which they analyze the Lyapunov stability of a simple spacecraft configuration for the closed-loop control of a rest-to-rest and a tracking maneuver.

Computational and Mathematical Papers

Many of the papers dealing with computational algorithms for the time-optimal control problem assume a bang-bang control profile and find the number of switches and the switch times for each controller. Some[37–40] also deal directly with the TPBVP, especially in nonlinear form, by iterating numerically for the missing initial conditions (costates). The two techniques used in these approaches are indirect (shooting methods) and direct methods. In the shooting methods,[37,38] several independent solution trajectories are determined numerically and the true set of initial conditions are generated from a linear combination of these solutions. Quasilinearization is used by several authors[39,40] when addressing nonlinear TPBVPs, but very often, the most difficult aspect of this method is determining a solution to use for the linearization process, as in the paper by Li and Bainum.[39] The initial costates, or Lagrange multipliers, are not specified in the boundary conditions, so Yeo et al.[40] explored how to choose these variables, in an optimal fashion, to achieve the fastest and most accurate convergence to the solution.

A related, and typical approach, is to obtain a single solution for a similar problem and modify the trajectories through subsequent iterations to include nonlinear effects and/or to improve the accuracy of the solution.[41–43] Roberts and Shipman[41] use a perturbation approach to solve a sequence of increasingly nonlinear problems. As is typical of perturbation approaches, this method uses the solution to the previous system as a forcing function for the next order in the expansion. A similar approach by Subrahmanyam[44] uses Newton's method to make successive approximations by linearizing the system through a discretization process. One algorithm in particular was frequently cited by later authors; the control algorithm, called switch time optimization (STO) was developed by Meier and Bryson.[45] This method was originally developed for a two-link robotic manipulator, but it is applicable to other dynamic systems as well. The STO method assumes that the controls are saturated for the entire maneuver and solves for the location of the finite number of switch times rather than the control magnitude. It uses the steepest descent algorithm in addition to an initial guess generated by the auxiliary cost functional

$$J = \int_0^{t_f} (1 - \boldsymbol{u}^T\boldsymbol{u})\,\mathrm{d}t \qquad (9)$$

One problem common to most of these methods is that they require a relatively good initial guess to converge to a solution, hence the use of a previous, linear solution or an assumed trajectory as a starting point. Conversely, direct methods such as the method of steepest descent operate directly in the function space of the performance index as in the paper by Storm.[42] This gradient projection method converges very quickly in the first few steps, but the rate of convergence drops rapidly as the desired accuracy of the solution increases. Therefore, its primary function is often to provide a good initial guess for use in other algorithms. Enright and Conway[46] developed a method for determining optimal orbit trajectories, using collocation and nonlinear programming. This method shows great promise for use in other optimal control applications because it appears to converge to within a desired accuracy in a reasonable amount of time, even with a relatively poor initial guess.

Early Solutions

The next set of papers to be discussed concern early attempts to solve the time-optimal attitude maneuver problem. The technology and analytical techniques available at the time these papers were written is quite different from the powerful algorithms available to today's researchers. Consequently, a priori assumptions commonly made in these papers may no longer be valid, and the solutions set forth in these papers cannot be considered state-of-the-art. As a group, however, they form the foundation for the next generation of solutions.

Quite a few of the early papers tackled the attitude control problem from a stabilization point of view.[47–51] If the angular displacements of a vehicle are small, the equations of motion may be linearized, thus allowing an analytical solution to be determined. A variety of control devices were considered, including active controls such as thrusters and control moment gyroscopes, and passive or, as one researcher put it, "semi-active" controls such as gravity gradient[51,52] and solar pressure torques.[53] In addition to restricting the problem to one of stabilization, where deviation from a nominal position is to be driven to zero, some researchers dealt only with the angular momentum of the spacecraft, thus avoiding the whole issue of Euler angles and auxiliary equations to determine the spacecraft attitude. The control systems in these papers drive the angular momentum to zero in some cases,[47,54,55] and to some nominal single-axis spin rate in others.[48,50] A recurring theme in most papers was the impracticality of implementing an optimal solution in real time due to the complexity of the problem and the time to compute a control input. This lead to solutions which were suboptimal[49,50,56] due to some assumptions about the form of the control profile. A few authors chose not to solve the strict time-optimal problem and used, for example, a cost functional which depended on the state and/or control vectors.[57]

Rigid-Body Solutions

The remainder of this survey is divided into four subtopics; rigid-body solutions, flexible spacecraft, specific configurations, and recent advances. The rigid-body solutions provide the basis for the flexibility studies. We begin our rigid-body discussion with the work of Akulenko and Lilov[58] which is very general and abstract. The angular velocity vector is restricted in such a way that an approximate solution for the control of a rigid body can be found. An open-loop LQR control problem is used to illustrate the technique. Chowdhry et al.[59,60] and Chowdhry and Cliff[61] explore the solution space of open-loop optimal control trajectories by first assuming no control over one of the angular velocity components. The control availability is varied by a perturbation approach, and the dependence of the solution on these parameters is explored. These papers, while developed for high performance aircraft, are applicable to spacecraft as well. Gorelov[62] chose to reorient the spin axis of a symmetric spacecraft. The equations were linearized under symmetry assumptions and through control of the magnitude of the angular velocity of the vehicle throughout the maneuver.

In two other very interesting papers, the authors use magnetic attitude controls in developing the models. In the first paper,[63] a single controller aligned with the spin axis of a spin-stabilized, symmetric spacecraft is used. The solution is found through an interactive graphical technique. The second paper[64] also uses a single controller on the spin axis and an assumed model of Earth's magnetic field; the complete open-loop, time-optimal, and energy-optimal solutions are generated and compared with control algorithms which were then currently in use.

Flexible Spacecraft

All of the papers in this section dealing with flexible spacecraft use a rigid hub with flexible appendages as a model, although the number of appendages and the type of controllers vary. All authors use the assumed vibrational modes of a cantilevered beam as the basis functions for discretizing the motion of the flexible components. In addition, the only maneuvers addressed are single-axis rotations about the axis of symmetry.

The simplest model was a rigid hub with a single flexible appendage with one control torque applied to the central hub. Ben-Asher et al.[65] found the solution to the linearized problem and used it as a nominal trajectory for the nonlinear system. They discovered that the open-loop maneuver time for the linear and nonlinear systems were similar, but the symmetry of the switching function was destroyed in the nonlinear example. Only one flexible mode was included in the numerical example. In another paper, Barbieri and Ozguner[66] address the flexible slewing problem through phase-plane analysis, but they also include just one flexible mode in the numerical example, and the resulting closed-loop control profile is bang-bang with multiple switches. Determination of the switch times for the closed-loop control of a single-axis, flexible spacecraft, using a single controller was presented by Dodds and Williamson.[67] The method is applicable to structures with a very low fundamental frequency, as is often the case for large space structures.

Meirovitch and Sharony[68] considered the same geometrical configuration, a rigid hub with one flexible appendage, but they added two force actuators at the hub, and a finite number of torque actuators (p of them) along the flexible appendage. The system was treated with perturbation methods, where the rigid-body solution corresponded to the zero-order equations, and the first-order equations contained the flexibility terms. The rigid-body model was subject to time-optimal control, resulting in a bang-bang solution. The flexibility effects were controlled using with an LQR algorithm using the maneuver time of the rigid-body solution as the final time in the first-order system. The LQR was augmented by an exponential term to drive the vibrational modes to rest as quickly as possible. Finally, the two solutions were applied simultaneously, with the bang-bang input produced by the hub actuator, and the p torque actuators applying the control input for the LQR system. Two additional papers, one by Meirovitch and Sharony and another by Meirovitch and Kwak, use a perturbation approach for closed-loop vibrational control of a flexible spacecraft. Time-optimal maneuvers are among those considered in both papers: the first paper is for a single-axis rotation[69] while the second extends the perturbation approach to multibody configurations.[70]

Thompson et al.[71] chose to avoid exciting the vibrational modes by smoothing the control input using a hyperbolic tangent function. The geometrical model was a rigid hub with four flexible appendages, located symmetrically around the hub. The system employed coarse (open-loop) and fine (closed-loop) controls, and the rise time of the control profile was governed by an independent parameter. The coarse control and the fine control were governed by different smoothing functions; the closed-loop system had a much smaller control magnitude than that allowed for the open-loop control to avoid problems with overshooting. Byers et al.[72] developed the fine control which was the only manner by which the flexible motion could be completely controlled and damped out, although the open-loop control did reduce the residual energy of the flexible elements by a significant factor. The results were compared with the bang-bang and LQR solutions, and several examples were presented that included tracking problems and multibody configurations. In addition to the analytical results, a related paper reported experimental results obtained with the smoothed control profile. The experimental apparatus consisted of a rigid hub with four flexible appendages, and the vibration suppression properties of the smoothed input were confirmed by the experiments.[12] Bang and Junkins[73] developed a near-minimum-time optimal control law by optimizing a stable, closed-loop control law with respect to design variables such as the smoothing parameter. The resulting profile has the optimal smoothing to minimize vibrational excitation and is near-time-optimal.

Singh and Vadali[74] have extended this work further by considering the three-dimensional motion of a hub with six flexible appendages arranged orthogonally about the spherical hub (each appendage is aligned with a $\pm$ axis of a body-fixed coordinate system). The authors have combined a Lyapunov controller with input shaping to produce a closed-loop, near-minimum-time controller that suppresses flexural motion much faster than previous methods.

The next three papers in this section deal with different aspects of a similar system, namely a rigid hub with N flexible appendages and m torques located along the appendages; a central torque is applied at the hub. The assumptions common in all three papers are that the control history is symmetric about the midpoint (mid-maneuver time) and the first n flexible modes are controlled. An error estimate was developed to give a conservative estimate of the number of modes which must be included to achieve a given accuracy. The open-loop control was found through a homotopy approach, and the resulting control profile was bang-bang with multiple switches. The first two papers[75,76] deal with the rest-to-rest maneuvers while the third[77] is a time-optimal, spin-up problem. Numerical examples of each type of maneuver are presented in detail.

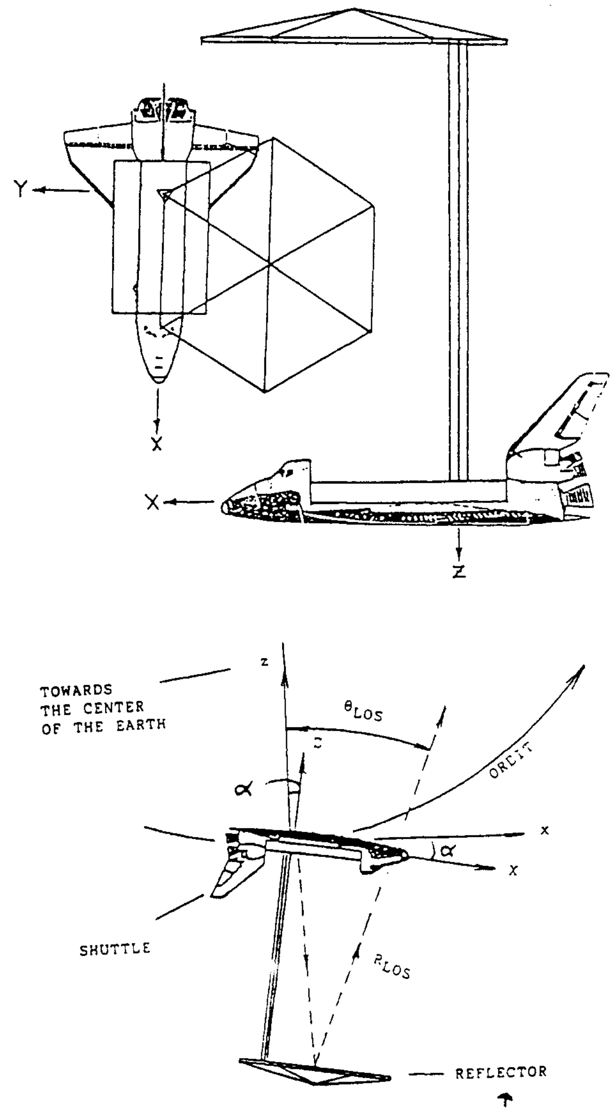

Fig. 1 SCOLE configuration (Ref. 86).

Bikdash et al.[78] combine minimum-time with minimum control effort to formulate a soft-constrained system. They develop a closed-loop strategy and examine its performance under disturbances to the system. Liu and Wie[79] approach the problem of a rest-to-rest, single-axis maneuver of a flexible spacecraft by addressing the issue of robustness with respect to the uncertainty in the modal parameters. They find that the smoothing approach clearly does not take advantage of all the available control energy.

Specific Configurations

Several papers concentrate on developing minimum-time maneuvers for very specific spacecraft configurations. Rajan[80] presents results for the Space Infrared Telescope Facility (SIRTF) for the small-angle slewing, or nod, maneuver. The first bending mode is included in the model, but due to small-angle arguments, the system is linearized, and the resulting closed-loop control profile is bang-bang with multiple switches. Two different investigations examine maneuvers for the Spacecraft Control Laboratory Experiment (SCOLE), which is a Shuttle-based array, shown in Fig. 1. Fisher et al.[81] assume a rigid-body behavior and develop the nonlinear closed-loop control problem for an arbitrary maneuver. The approach taken for finding the trajectory is somewhat unusual in that the optimal axis for rotation is chosen prior to the maneuver. This optimal axis is assumed to be perpendicular to the line of sight of the array, and is dictated by minimum inertia. The control is then bang-bang with terms added to cancel gyroscopic effects

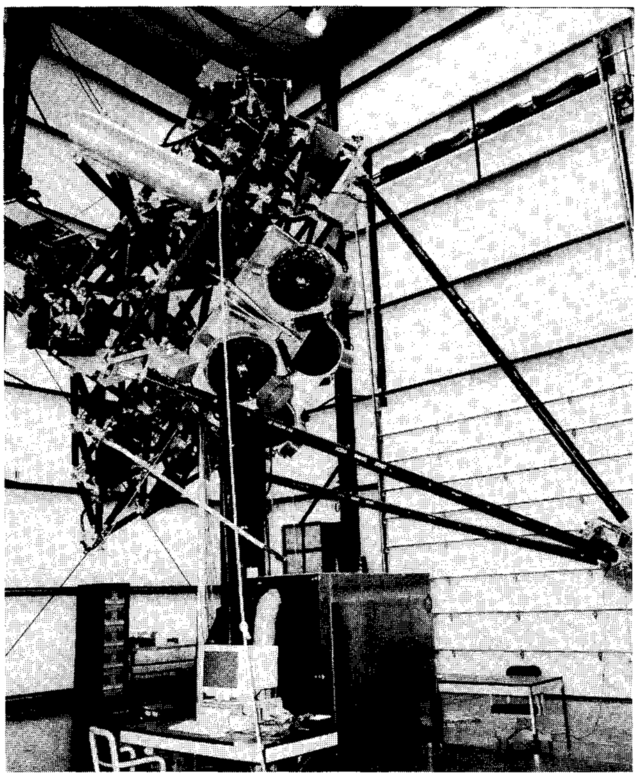

Fig. 2 ASTREX test article.

which occur when rotating about a nonprincipal axis. Bainum and Li[82] treat the SCOLE array as flexible and also compute the open-loop control algorithm for an arbitrary maneuver. Their strategy is to formulate the problem as an LQR, and through quasilinearization, solve the resulting TPBVP. The final time is iteratively reduced until a bang-bang profile is obtained. The last specific configuration discussed in this survey is the Advanced Structures Technology Research Experiment (ASTREX), which is an Earth-based testbed for experiments in control of large space structures. This facility, shown in Fig. 2, has been fully operational since 1991, and research publications have now begun to appear. One of the first papers to include experimental data[83] addressed near-minimum-time maneuvers. The control profile is smoothed and the switch times are computed by a variety of methods, including the STO, estimation of linear velocity interval switching (ELVIS), and closed-loop parabolic switching functions. A much more recent paper by Vadali et al. includes the effects of gravitational loading, complex structural connections, and coasting arcs. The approach was to develop an open-loop near-minimum-time solution using control smoothing and fuel consumption constraints; the solution was accomplished by applying the sequential quadratic programming method.

Recent Advances

The final group of publications examined in this survey include the most recent advances in solving the full three-dimensional problem. Although papers in the previous section address the solutions of specific problems, a general numerical method for solving the three-dimensional control problem of a general rigid or flexible spacecraft has not yet appeared. Most investigators use Euler parameters to describe the attitude of the spacecraft in order to avoid any numerical singularities, and quite a few authors suggest that the Euler Rotation be used as a nominal solution because this maneuver traces out the shortest path through space.

In a very recent presentation, Li and Bainum[85] assumed that even if the time-optimal solution were not an Euler Principal Rotation, then the time-optimal path should at least lie close to the principal rotation path. The problem was then posed as an open-loop, minimum-control-effort optimal control problem; through quasilinearization combined with the method of particular solutions, new control profiles were determined by iteratively reducing the maneuver time until the solution failed to converge. The maneuver time obtained in this fashion was taken as the minimum time for the maneuver. It should be noted that the modified control profiles obtained in this manner drive the spacecraft along a path which is significantly different from the Euler axis rotation. Several example reorientations of the rigid SCOLE model were presented in related papers.[40,85–87]

Etter[88] developed the solution to the time-optimal Euler axis rotation for an asymmetric spacecraft. For a given maneuver, the system reduces to a single degree of freedom. Using the reduction in order and the assumption that at least one controller must be saturated at all times, a closed-form solution for the open-loop control algorithm was determined. This approach is time-optimal but the motion is constrained to be an Euler axis rotation. Therefore, the result is time-optimal only along the specified path. Several numerical examples were presented.

Tan et al.[89] used a shallow, spherical shell as a spacecraft model. Through quasilinearization and the method of particular solutions, the near-minimum-time solution for the open-loop system was found by solving an LQR iteratively, reducing the final time until a bang-bang control profile was obtained.

Bilimoria and Wie[90] show that the Euler axis rotation is not, in general, the time-optimal solution. For a model, they used the simplest geometrical configuration: a sphere with three orthogonal controllers. With the spherical symmetry, Euler's equations completely uncouple and it might be expected that the Euler axis rotation would be a time-optimal solution. The structure of the control profile was examined and it was rigorously shown that at all times at least one controller must be saturated. A good guess for the control profile was found by using a parameter to make successive modifications on the control constraint. This control logic and resulting trajectory were used as the initial guess in the multiple shooting algorithm. The final open-loop solution was bang-bang with multiple switches; the resulting trajectory was significantly different from, and 8.5% faster than, the Euler axis rotation for the same reorientation.

Several aspects of the resulting motion bear discussion here, mainly because they contradict several assumptions which have been used to compute time-optimal solutions. To illustrate this point, it had been assumed that a 180-deg rotation about one body-fixed controller axis (u_3, for example) would result in a bang-bang profile for u_3 with neither of the other two controllers making any contribution to the motion; the resulting motion would be a planar rotation about the inertial three-axes. Bilimoria and Wie show that the true time-optimal solution has a significant nutational component, creating an angle of 45 deg at midmaneuver between the body-fixed three-axis and the inertial three-axis. The net torque about the inertial axis for this maneuver is shown to be significantly greater than the saturated value of the u_3 controller by itself; the u_1 and u_2 controllers increase the turning power of the spacecraft because the predominant spin axis has nutated away from the inertial axis. This maneuver clearly does not minimize the control effort, because a significant portion of the control input goes toward canceling the nutation at the end of the maneuver. This explains why the approach of iteratively reducing the final time of a minimum-control-effort problem does not yield a true time-optimal solution.

In a more recent extension of their work, Bilimoria and Wie[91] have applied their method to an axisymmetric body. They used nondimensional equations of motion such that the inertia and torque ratios were specified by two independent parameters. This provided a simple means of investigating various shapes from predominately oblate to predominately prolate. The results show a marked decrease in maneuver time when nutational motion is allowed.

Seywald and Kumar[92] also investigated open-loop, minimum-time maneuvers of an inertially symmetric spacecraft. They included finite- and infinite-order, singular-arc control profiles; they also proved that principal axis rotations are not the optimal solution and they demonstrated that infinite-order singular controls are optimal for certain problems.

Byers and Vadali[93] built upon the work of Bilimoria and Wie in the sense that they reproduce the results reported there, but the focus of their paper was to develop a control algorithm which could be implemented in real time. The model was a rigid spacecraft, and Euler's equations were then linearized by assuming that the nonlinear inertia terms were dominated by the control forces. Although linear, the resulting system does not decouple. A discussion of switch times for time-optimal maneuvers was included, and results were computed using the STO algorithm. The authors used simplifying approximations to reduce the computation time. An approximation to the state transition matrix was developed, and the degree of approximation was discussed in detail. The ELVIS algorithm was used to compute switch times, and the result was compared to the STO solution. Due to terminal errors which occur while using the ELVIS algorithm, an additional set of closed-loop, parabolic switching functions were used as the system approached the origin. This approximate solution compared very favorably with the STO solution, and had a significantly lower computational time.

Conclusions

Time-optimal control problems, particularly as applied to spacecraft attitude maneuvers, are complex and generally difficult to solve. Researchers continue to explore different aspects of the problem, particularly the areas of flexibility effects, specific spacecraft configurations, and solutions to the general three-dimensional reorientation of a rigid body. The scope of the papers in this survey and the number of different approaches taken in them are a clear indication that this is a broad-based, multidisciplinary field, requiring expertise in a number of areas to obtain rigorous yet practical solutions. However, the results obtained are encouraging, and it is evident that researchers continue to meet the challenges posed by the time-optimal reorientation problem. As the current aspects of the problem are successfully addressed, new facets and applications open up for exploration.

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